Abstracts

3-Connected Minor Minimal Non-Projective Planar Graphs with an Internal 3-Separation

Arash Asadi, Georgia Institute of Technology

The property that a graph has an embedding in the projective plane is closed under taking minors. So by the well known theorem of Robertson and Seymour, there exists a finite list of minor-minimal graphs, call it L, such that a given graph G is projective planar if and only if G does not contain any graph isomorphic to a member of L as a minor. Glover, Huneke and Wang found 35 graphs in L, and Archdeacon proved that those are all the members of L. In this talk we show a new strategy for finding the list L. Our approach is based on conditioning on the connectivity of a member of L. Assume G is a member of L. If G is not 3-connected then the structure of G is well understood. In the case that G is 3-connected, the problem breaks down into two main cases, either G has an internal separation of order three or G is internally 4-connected. In this talk we find the set of all 3-connected minor minimal non-projective planar graphs with an internal 3-separation. This is joint work with Luke Postle and Robin Thomas.

Forcing Large Transitive Subtournamets

Maria Chudnovsky, Columbia University

The Erdos Hajnal Conjecture states roughly that a graph with some induced subgraph excluded has a large clique or a large stable set. A similar statement can be formulated for tournaments (a tournament is an orientation of a complete graph), replacing cliques and stable sets by transitive subtournaments; and the two conjectures turn out to be equivalent. This talk will survey a number of recent results related to the latter conjecture. In particular, we will discuss a new infinite class of tournaments excluding which forces large transitive subtournaments; to the best of our knowledge this is the first such class not obtained by the so-called substitution operation.

Refinements of Archdeacon's Theorem

Guoli Ding, Louisiana State University

Archdeacon characterized projective planar graphs by 35 forbidden minors. In this talk, we discuss some refinements of this result.

Nonrepetitive Colourings

Vida Dujmović, McGill University, Canada

A vertex colouring of a graph is nonrepetitive if there is no path whose first half receives the same sequence of colours as the second half. This talk will present some results on nonrepetitive colourings of planar graphs and graphs of bounded degree.

Coloring Graphs on Surfaces

Zdenek Dvorak, Charles University, Czech Republic

We give an overview of the recent progress on coloring embedded graphs.

Extremal Results in Sparse Pseudorandom Graphs

Jacob Fox, Massachusetts Institute of Technology

Szemerédi's regularity lemma is a fundamental tool in extremal combinatorics. However, the original version is only helpful in studying dense graphs. In the 1990s, Kohayakawa and Rödl proved an analogue of Szemerédi's regularity lemma for sparse graphs as part of a general program toward extending extremal results to sparse graphs. Many of the key applications of Szemerédi's regularity lemma use an associated counting lemma. In order to prove analogues of these results for sparse graphs, it remained a well-known open problem to prove a counting lemma in sparse graphs.

The main advance in this talk lies in a new counting lemma, proved following the functional approach of Gowers, which complements the sparse regularity lemma of Kohayakawa and Rödl, allowing us to count small graphs in regular subgraphs of a sufficiently pseudorandom graph. We use this to prove analogues of several well-known combinatorial theorems, including the graph removal lemma, the Erdős-Stone-Simonovits theorem, and Ramsey's theorem, for subgraphs of sparse pseudorandom graphs. These results extend and improve upon a substantial body of previous work.

Joint work with David Conlon and Yufei Zhao.

Growth-Rates of Matroid Classes

Jim Geelen, University of Waterloo, Canada

For a minor-closed class of matroids, we consider the maximum number, $h(n)$, of elements in a simple rank-n matroid in the class. This growth-rate function, when finite, is known to be either linear, quadratic, or exponential. For exponentially dense classes we determine $h(n)$ precisely, for sufficiently large n, and also describe the extremal matroids in the class. This is joint work with Peter Nelson.

Thin Spanning Trees

Michel Goemans, Massachusetts Institute of Technology

A spanning tree T in a graph G is ϵ-thin if T contains at most an ϵ-fraction of the edges of every cut. Goddyn's conjecture claims that every $f(\epsilon)$-edge-connected graph contains an ϵ-thin tree for a suitable function f. A positive resolution of this conjecture would imply a constant factor approximation algorithm for the asymmetric travelling salesman problem, and an alternate proof of the recent result of Thomassen on the existence of nowhere zero 3-flows in sufficiently edge-connected graphs.

In this talk, we will discuss several variants of Goddyn's conjecture. In particular, we will show that spectral graph sparsification techniques proves a weakening of the conjecture in which the spanning tree requirement is replaced by simply having as many edges, a question raised by Thomasse. In this case, the edge-connectivity requirement can be weakened to having at least $c(\epsilon)n$ edges. Among other results, we also show that thin spanning trees exist in $\log(n)/\log\log(n)$-edge connected graphs.

Multi-flow and Algorithm

Bertrand Guenin, University of Waterloo, Canada

Geelen and Guenin proved that given a signed graph without odd-$k5$ minor and Eulerian weights, the cut-condition is sufficient for the existence of an integer multi-flow. We are interested in the following related question. Given an arbitrary signed graph with eulerian weights, can we find in polynomial time one of the following: an integer multi-flow, a cut that violates the cut condition, or an odd-$K5$ minor? This is joint work with Leanne Stuive.

On Matchings and Covers

Penny Haxell, University of Waterloo, Canada

Let H be a hypergraph. A *matching* in H is a set of pairwise disjoint edges of H. A *cover* of H is a set C of vertices that meets all edges of H. We discuss a number of conjectures and results that bound the minimum size of a cover of H in terms of the maximum size of a matching in H, for various classes of hypergraph H. For example, we consider r-partite hypergraphs and triangle hypergraphs.

On Tractability of Minimum 0-Extension Problems

Hiroshi Hirai, University of Tokyo, Japan

Given an undirected graph G, a set V including $V(G)$, and a nonnegative cost function c on the set of all pairs on V, The minimum 0-extension problem for G, V, c is to find a 0-extension d of d_G with $\sum_x yc(xy)d(x, y)$ minimum, where a 0-extension is a metric d on V extending d_G such that for every x in V there exists s in $V(G)$ with $d(x, s) = 0$. This problem, introduced by A. Karzanov in 1998, includes minimum $s-t$ cut problem for $G = K_2$ and, more generally, multiway cut problem for $G = K_n$.

Our interest is a question raised by Karzanov: What is the graph G for which the minimum 0-extension problem on (fixed) G is polynomially solvable? In this talk, we describe some aspects and new results to this problem.

Testing Graph MSO Properties: Has it all been told already?

Petr Hliněný, Masaryk University, Czech Republic

Despite more than 20 years passed since the publication of famous Courcelle's theorem about solvability of MSO2 definable graph properties in parameterized linear time on graphs of bounded tree-width, some interesting questions in this regard remained without answers. Noticeably, can one provide such an algorithm with elementary dependence of its runtime on the MSO2 formula size? This is generally impossible (unless $P = NP$) even in the class of all trees by a 2004 result of Frick and Grohe. Yet, we prove that MSO2 properties can be tested in parameterized linear time, with elementary dependence on the formula, on graphs of bounded tree-depth. We also provide an analogical result for MSO1 properties. Our results widely generalize recent papers by Lampis (ESA 2010) and Ganian (IPEC 2011).

Joint work with my student Jakub Gajarský.

Pfaffian Orientations, Symmetric Matrices, and Linear Programming

Satoru Iwata, Kyoto University, Japan

Robertson, Seymour, and Thomas (1999) presented a polynomial algorithm for determining if a given bipartite graph admits a Pfaffian orientation, resolving a long-standing open problem posed by Polya in 1913. This problem was known to have a variety of equivalent formulations in terms of directed circuits and sign-solvability of linear equations.

In this talk, we review recent applications of their result in qualitative matrix analysis and mathematical programming. In particular, we discuss the notion of sign-solvability of linear programming and linear complementarity problems.

This talk is based on joint works with Naonori Kakimura.

Excluded Forest Minors and the Erdos-Posa Property

Gwenaël Joret, Université Libre de Bruxelles, Belgium

A classical result of Robertson and Seymour states that the set of graphs containing a fixed planar graph H as a minor has the so-called Erdos-Posa property; namely, there exists a function f depending only on H such that, for every graph G and every positive integer k, either G has k vertex-disjoint subgraphs each containing H as a minor, or there exists a subset X of vertices of G of size at most $f(k)$ such that $G - X$ has no H-minor. While the best function f currently known is super-exponential in k, a $O(k \log k)$ bound is known in the special case where H is a forest. This is a consequence of a theorem of Bienstock, Robertson, Seymour, and Thomas on the pathwidth of graphs with an excluded forest-minor. In this talk I will sketch a proof that the function f can be taken to be linear when H is a forest. This is best possible in the sense that no linear bound exists if H has a cycle.

Joint work with S. Fiorini and D. R. Wood.

On the Connectivity of Random Graphs from Addable Classes

Mihyun Kang, Technische Universität Graz, Austria

A class $\mathcal{G}$ of graphs is called *weakly addable* if for any $G \in \mathcal{G}$ and any two distinct components C_1 and C_2 in G, any graph that can be obtained by adding an edge between C_1 and C_2 is also in *mathcalG*. McDiarmid, Steger and Welsh conjectured [Random graphs from planar and other addable classes. In Topics in discrete mathematics, volume 26 of Algorithms Combin., pages 231–246. Springer, 2006] that a random graph on n vertices from $\mathcal{G}$ is connected with probability at least $e^{-1/2} + o(1)$, as $n \to \infty$. We prove this conjecture under the additional assumption that $\mathcal{G}$ is *monotone*, i.e., if for any $G \in \mathcal{G}$ and any edge $e \in G$, the graph G without e is also in $\mathcal{G}$. This is joint work with Konstantinos Panagiotou.

Coloring 3-Colorable Graphs; Graph Theory Finally Strikes Back!

Ken-ichi Kawarabayashi, National Institute of Informatics, Japan

We consider the problem of coloring a 3-colorable graph in polynomial time using as few colors as possible.

Starting with Wigderson in 1982 ($O(n^{1/2})$ colors), Blum in 1990 came with the first polynomial improvements ($O(n^{3/8})$ colors). His improvement is based on graph theoretical approach.

Karger, Motwani, Sudan in 1994 is the first to use semi-definite programming (SDP) to give improvement, and then Karger and Blum in 1997 combines Blum's method with the SDP improvement to show that $O(n^{0.2142})$ colors suffices.

Since then, the only improvements in semi-definite programming have been made (Arora, Chlamtac, and Charikar in 2006 ($O(n^{0.2111})$ colors), and Chlamtac in 2007 ($O(n^{0.2072})$ colors)).

We present the first improvement on the graph theory side since Blum in 1990. With a purely graph theoretical approach, we get down to $O(n^{4/11})$ colors (over Blum's $O(n^{3/8})$ colors). Combining it with SDP, we get down to $O(n^{0.2038})$ colors.

Joint work with Mikkel Thorup (AT&T Research)

A Refinement of the Triangle Version of the Corradi-Hajnal Theorem

Alexandr V. Kostochka, University of Illinois at Urbana-Champaign

An important part of the Corradi-Hajnal Theorem says that if $n = 3k$, then every n-vertex graph G with minimum degree at least $2k = 2n/3$ contains k vertex-disjoint triangles. The restriction on the minimum degree is sharp. This is equivalent (by switching to the complement) to the statement that every n-vertex graph H with maximum degree at most $k-1$ has an *equitable* k-coloring, that is a proper coloring of vertices of H with k colors such that the sizes of color classes differ by at most 1. In 1970, Hajnal and Szemerédi generailzed this result by proving the conjecture of Erdős that every graph with maximum degree at most r has an equitable $r+1$-coloring. In this talk, we prove a Brooks-type result describing for $r \geq n/4$ all n-vertex graphs with maximum degree at most r that do not admit an equitable r-coloring. In particular, we conclude that for $k \geq 6$, at most one $3k$-vertex graph with minimum degree at least $2k-1$ and independence number at most k does not contain k vertex-disjoint triangles. This is joint work with H. A. Kierstead.

Deciding Properties in Sparse Combinatorial Structures

Daniel Král, Charles University, Czech Republic

Algorithmic metatheorems guarantee that certain types of problems have efficient algorithms. A classical example is the result of Courcelle asserting that every MSOL property can be tested in linear time for graphs with bounded tree-width. We focus on simpler properties, those expressible in first order logic (FOL); our main result asserts that every FOL property can be decided in linear time for graphs in a class with bounded expansion. Such classes include graphs with bounded maximum degree or proper-minor closed classes of graphs.

The talk is based on joint work with Zdenek Dvorak and Robin Thomas.

Minors of Partially Embedded Graphs

Jan Kratochvil, Charles University, Czech Republic

In Angelini et al [SODA 2010] we have presented a linear time algorithm to test planarity of partially embedded graphs. Now we present a characterization by a finite family of forbidden configurations which we call pems (partially embedded minors). For some of them, their presence can be tested in polynomial time, but in general this question - which would conclude the parallela to Graph Minors - is open. The presentation is based on joint work with Vit Jelinek and Iganz Rutter presented at SoCG 2011.

Linkless Embeddings in 3-space

Stephan Kreutzer, Technical University Berlin, Germany

We consider piecewise linear embeddings of graphs in the 3-space R^3. Such an embedding is "linkless" if every pair of disjoint cycles forms a trivial link (in the sense of knot theory). Robertson, Seymour and Thomas showed that a graph has a linkless embedding in R^3 if and only if it does not contain as a minor any of seven graphs in Petersen's family (graphs obtained from K_6 by a series of $Y\Delta$ and ΔY operations). They also showed that a graph is linklessly embeddable in R^3 if and only if it admits a "flat embedding" into R^3.

In this talk we present an algorithm running in time n^2 which, given a graph G as input either determines that G contains a graph of the Petersen family as minor, and hence is not linklessly embeddable, or computes a flat embedding of G. We also briefly discuss recent extension of this algorithm running in linear time.

An Upper Bound on the Fractional Chromatic Number of Triangle-Free Subcubic Graphs

Chun-Hung Liu, Georgia Institute of Technology

An $(a : b)$-coloring of a graph G is a function f which maps the vertices of G into b-element subsets of some set of size a in such a way that $f(u)$ is disjoint from $f(v)$ for every two adjacent vertices u and v in G. The fractional chromatic number $\chi_f(G)$ is the infimum of a/b over all pairs of positive integers a, b such that G has an $(a : b)$-coloring. Heckman and Thomas conjectured that the fractional chromatic number of every triangle-free graph G of maximum degree at most three is at most 2.8. Hatami and Zhu proved that $\chi_f(G) \leq 3 - 3/64 \approx 2.953$. Lu and Peng improved the bound to $\chi_f(G) \leq 3 - 3/43 \approx 2.930$. Recently, Ferguson, Kaiser and Král proved that $\chi_f(G) \leq 32/11 \approx 2.909$. In this talk, I will prove that $\chi_f(G) \leq 43/15 \approx 2.867$.

Extending Polynomials in Combinatorics

Martin Loebl, Charles University, Czech Republic

Tutte polynomial is an extension of the chromatic polynomial. Fruitful source of extensions is q-commutation and determinant: q-MacMahon Master theorem is an important example. I will describe the basic setting and show application in other parts of combinatorics.

Extremal Graphs and Graph Limits

László Lovász, Eötvös Loránd University, Hungary

Growing sequences of dense graphs have a limit object in terms of a symmetric measurable 2-variable function. A typical use of this fact in graph theory is the following: we want to prove a result, say an inequality between subgraph densities. We look at a sequence of counterexamples, and consider their limit. Often this allows clean formulations and arguments that would be awkward or impossible in the finite setting. We illustrate this by some results on Sidorenko's conjecture and "common graphs."

This setting also allows us to pose and in some cases answer general questions about extremal graph theory: which inequalities between subgraph densities are valid, and what is the possible structure of extremal graphs.

Extremal Problems in Eulerian Digraphs

Jie Ma, University of California, Los Angeles

Graphs and digraphs behave quite differently and many natural results for graphs are often trivially false for general digraphs. It is usually necessary to consider restricted families of digraphs to obtain meaningful results. One such very natural family is Eulerian digraphs, i.e., digraphs in which in-degree of every vertex equals its out-degree.

We will discuss several natural parameters for Eulerian digraphs and study their connections. In particular, we show that for any Eulerian digraph with n vertices and m arcs, the minimum feedback arc set (that is a smallest set of arcs whose removal makes the digraph acyclic) has size at least $m^2/2n^2 + m/2n$, and this bound is optimal. Using this result we show how to find subgraphs of high minimum in- and out-degree and also long cycles in Eulerian digraphs. These results were motivated by a conjecture of Bollobas and Scott. Joint work with Huang, Shapira, Sudakov and Yuster.

Structure Theorem and Isomorphism Test for Graphs with Excluded Topological Subgraphs

Dániel Marx, Hungarian Academy of Sciences, Hungary

We generalize the structure theorem of Robertson and Seymour for graphs excluding a fixed graph H as a minor to graphs excluding H as a topological subgraph. We prove that for a fixed H, every graph excluding H as a topological subgraph has a tree decomposition where each part is either "almost embeddable" to a fixed surface or has bounded degree with the exception of a bounded number of vertices. Furthermore, we prove that such a decomposition is computable by an algorithm that is fixed-parameter tractable with parameter $|H|$.

We show that on graphs excluding H as a topological subgraph, Graph Isomorphism can be solved in time $n^{f(H)}$. This result unifies and generalizes two previously known important polynomial-time solvable cases of Graph Isomorphism: bounded-degree graphs and H-minor free graph. The proof of this result needs a generalization of our structure theorem to the context of invariant treelike decomposition.

Discrepancy, Bansal's Algorithm and the Determinant Bound

Jiri Matousek, Charles University, Czech Republic

Recently Nikhil Bansal found a polynomial-time algorithm for computing low-discrepancy colorings, based on semidefinite programming. It makes several existential bounds for the discrepancy of certain set systems, such as all arithmetic progressions on $\{1, 2, \ldots, n\}$, constructive, which has been a major open problem in discrepancy theory. We use Bansal's result, together with the duality of semidefinite programming and a linear-algebraic lower bound for discrepancy due to Lovasz, Spencer, and Vesztergombi, to answer an old question of Sos, concerning the maximum possible discrepancy of the union of two set systems.

Clique Immersion in Digraphs

Jessica McDonald, Simon Fraser University, Canada

Immersion is a containment relation between graphs (or digraphs) which is defined similarly to the more familiar notion of minors, but is incomparable to it. In this talk we focus on immersing a bidirected clique $\vec{K}_t$ in an Eulerian digraph. We show that every Eulerian digraph with minimum degree at least $t(t-1)$ immerses $\vec{K}_t$, and for $t = 4$ this requirement can be lowered to $t - 1$. We also provide a rough structure theorem for $\vec{K}_t$-immersion in Eulerian digraphs. Joint work with Matt DeVos, Bojan Mohar and Diego Scheide.

Large Clique Minors in Vertex Transitive Graphs

Bojan Mohar, Simon Fraser University, Canada

An important (yet unpublished) theorem of Babai states that there exists a function $f : N \to N$ so that every finite vertex transitive graph without K_n as a minor is either $(f(n), f(n))$-ring-like or is a vertex transitive map on the torus. A relatively short proof of Babai's theorem with explicit values for $f(n)$ will be presented. This talk is a joint work with Matt DeVos.

Intersection Theorems for Finite Sets

Dhruv Mubayi, University of Illinois at Chicago

Finite extremal set theory is concerned with the following general problem: Suppose we have a collection F of subsets of an n-element set and we have some restriction on the possible intersection sizes of pairs of sets in F. What is the maximum number of subsets that F can contain? Surprisingly, solutions to various special cases of this problem have deep implications in many other areas, including coding theory, geometry, and computer science. A particular famous example is due to Frankl and Rödl, who solved a 250-dollar problem of Erdös by proving that if n is a multiple of 4 and $n/4$ is excluded as an intersection size, then $|F| < (1.99)^n$. We extend this result by showing that if some additional (rather mild) restrictions are placed on the possible intersection sizes, then $|F| < (1.63)^n$. This is joint work with Vojtěch Rödl.

Subgraph Statistics for Geometric Graphs

Jaroslav Nesetril, Charles University, Czech Republic

We determine the asymptotics logorithmic density of subgraphs of large geometric graphs and of some large sparse graphs (from a nowhere dense class). This also leads to a unified approach to graph limits for both sparse and dense classes. Joint work with Patrice Ossona de Mendez, Paris.

Graphs of Small Rank-Width are Pivot-Minors of Graphs of Small Tree-Width

Sang-il Oum, Korea Advanced Institute of Science and Technology, South Korea

We prove that every graph of rank-width k is a pivot-minor of a graph of tree-width at most $2k$. We also prove that graphs of rank-width at most 1, also known as distance-hereditary graphs, are exactly vertex-minors of trees, and graphs of linear rank-width at most 1 are precisely vertex-minors of paths. This is a joint work with O-Joung Kwon.

Unavoidable Minors of Large 2- and 3-Connected Matroids

James Oxley, Louisiana State University

It is well known that every sufficiently large 2-connected loopless graph has a big cycle or a big bond. Twenty years ago, at the Seattle Graph Minors Conference, Robin Thomas asked whether the analogous result is true for 2-connected matroids. During that meeting, Schrijver, Seymour, and Lovasz answered the question affirmatively. This talk will survey results in this area, the most recent of which relate to capturing small sets of elements in large unavoidable minors.

5-List-Coloring Graphs on Surfaces

Luke Postle, Georgia Institute of Technology

Thomassen proved that there are only finitely many 6-critical graphs embeddable on a fixed surface. He also showed that planar graphs are 5-list-colorable. We develop new techniques to prove a general theorem for 5-list-coloring graphs embedded on a fixed surface.

Namely, for every surface S and every integer $C > 0$, there exists D such that the following holds: Let G be a graph embedded in a surface S with edge-width at least D and a list assignment L such that, for every vertex v in G, $L(v)$ has size at least five. If F is a collection of any number of facial cycles of length at most C such that every two cycles in F are distance at least D apart and every cycle in F has a locally planar neighborhood up to distance $D/2$, then any proper L-coloring of F, such that for every cycle in F the coloring of that cycle extends to a coloring of its neighborhood of distance $100 \log C$, extends to an L-coloring of G.

This theorem implies the following results. In what follows, let S be a fixed surface, G be a graph embedded in S (except in 4, where G is drawn in S) and L a list assignment such that, for every vertex v of G, $L(v)$ has size at least five.

1. If G has large edge-width, then G is 5-list-colorable. (Devos, Kawarabayashi and Mohar)
2. There exists only finitely many 6-list-critical graphs embeddable in S. (Conjectured by Thomassen, Proof announced by Kawarabayashi and Mohar) As a corollary, there exists a linear-time algorithm for deciding 5-list-colorability of graphs embeddable on S. Furthermore, we exhibit an explicit bound on the size of such graphs.
3. There exists $D(S)$ such that the following holds: If X is a subset of the vertices of G that are pairwise distance at least $D(S)$ apart, then any L-coloring of X extends to an L-coloring G. For planar graphs, this was conjectured by Albertson and recently proved by Dvorak, Lidicky, Mohar, and Postle.
4. There exists $D(S)$ such that the following holds: If G is a graph drawn in S with face-width at least $D(S)$ such that any pair of crossings is distance at least D apart, then G is L-colorable. For planar graphs, this was recently proved by Dvorak, Lidicky and Mohar.

Joint work with Robin Thomas.

Separators, Brambles, Tree Decompositions and Excluding Clique Minors

Bruce Reed, McGill University, Canada

The talk surveys the relationships between these topics highlighting the fundamental contributions of Robin Thomas.

Embedding Continuua into a Surface

Bruce Richter, University of Waterloo, Canada

In this joint work with Robin Christian, we prove the following theorem.

Let M be a locally connected, compact, metric space and let S be a surface. Then M embeds in S if and only if M does not contain one of:

(i) a generalized thumbtack;

(ii) a surface of smaller genus than that of S; or

(iii) one of the finitely many finite graphs that (topologically) minimally do not embed in S.

Topologically Well-quasi-ordered Classes of Graphs

Neil Robertson, Ohio State University

When is a topologically closed class T of finite graphs a well-quasi-order (wqo)? We will define when T is short chained and when it is nearly subcubic, and note that being a wqo implies being short chained and being nearly subcubic implies being a wqo. Thus the implication from short-chained to nearly subcubic would show these three properties are equivalent. We intend to systematically apply this approach to three elementary kinds of topologically closed classes, namely minor closed, immersion closed and valency bounded, to try to prove that short chained implies nearly subcubic. The obvious first case to test our conjectures on is series-parallel networks, where many of the difficulties already appear.

On Subgraphs of Large Girth

Vojtěch Rödl, Emory University

Embeddability of Infinite Graphs

Gelasio Salazar, Universidad Autonoma de San Luis Potosi, Mexico

Robertson and Seymour proved that there is a function $f(g)$ tending to infinity so that, if a graph G does not embed in any surface of Euler characteristic at least $2-2g$, then G has one of the following as a minor:

1. $f(g)$ disjoint copies of either $K_{3,3}$ or K_5;
2. $f(g)$ copies of either $K_{3,3}$ or K_5 that are disjoint except for a common vertex;
3. $f(g)$ copies of either $K_{3,3}$ or K_5 that are disjoint except for two common vertices; or
4. $K_{3,f(g)}$.

We have proved the following extension to infinite graphs:

A countable graph embeds in some orientable surface if and only if it does not contain as a minor one of the following graphs:

1. infinitely many disjoint copies of either $K_{3,3}$ or K_5;
2. infinitely many copies of either $K_{3,3}$ or K_5 that are disjoint except for a common vertex;
3. infinitely many copies of either $K_{3,3}$ or K_5 that are disjoint except for two common vertices; or
4. $K_{3,\aleph_0}$.

This is joint work with Robin Christian (Hamburg) and Bruce Richter (Waterloo).

On Limits of Edge-Coloring Models

Alexander Schrijver, Centrum Wiskunde & Informatica, Netherlands

Graph limits can be considered equivalently as limits of vertex-coloring models. We discuss that similarly limits of edge-coloring models exist and that they also form a compact space. Joint work with Guus Regts.

Induced Subgraphs and Subtournaments

Paul Seymour, Princeton University

Let H be a graph and let I be the set of all graphs that do not contain H as a minor. Then H is planar if and only if there is an upper bound on the treewidth of the members of I; and there are many other similar theorems that relate properties of H to structural properties of the members of I. Let us call them "structure theorems".

What about structure theorems for other containment relations instead of minor containment? For induced subgraph containment, there are virtually no such theorems known, but for subtournament containment there are some quite pretty structure theorems, mostly excluding tournaments that are almost transitive.

A transitive tournament is the tournament analogue of both a stable set and a clique, so what happens if we exclude as induced subgraphs TWO graphs instead of one; one almost a stable set, and the other almost a clique? This turns out to be a better problem with some nice answers.

Partly joint work with Maria Chudnovsky, Sasha Fradkin, Ilhee Kim, Gaku Liu, Sergei Norin, Bruce Reed.

Tutte's Three-Edge-Colouring Conjecture

Paul Seymour, Princeton University

The four-colour theorem is equivalent to the statement that every planar cubic graph with no cut-edge is 3-edge-colourable. What about non-planar cubic graphs? The Petersen graph is not 3-edge-colourable, and in 1966 Tutte conjectured that every cubic graph with no cut-edge that does not contain the Petersen graph as a minor is 3-edge-colourable.

In 1996 we proved this conjecture, but did not publish the result, for reasons that escape me. We are currently getting it all back together for publication; and this talk is an outline.

A graph is “apex” if deleting some vertex makes it planar; and “doublecross” if it can be drawn in the plane with crossings, but with only two crossings and both incident with the same region. Apex and doublecross cubic graphs do not have Petersen minors.

Our proof falls into three parts:

(a) Proving that any minimal counterexample to Tutte’s conjecture is “theta-connected” (vaguely, a cubic graph is theta-connected if every small edge-cut has a small number of vertices on one side); this part we did publish in 1996.

(b) Proving that every theta-connected cubic graph with no Petersen minor is either apex or doublecross, except for one twenty-vertex graph; this is the main part of the lecture.

(c) Proving that apex and doublecross graphs with no cut-edges are three-edge-colourable; this requires modifying the computer proof of the four-colour theorem.

Joint work with Neil Robertson and Robin Thomas.

From Online Colouring to Online Vector Bin Packing

Bruce Shepherd, McGill University, Canada

We discuss an older lower bound on the performance ratio for online colouring. We then show how it yields a lower bound for online vector bin packing for bins with capacity B. The bounds are tight for $B = 1$, but suggest several interesting upper bound questions (for colouring and bin packing).

Joint work with Y. Azar, S. Kamara.

Clique Minors and Odd Clique Minors in Graphs

Zi-Xia Song, University of Central Florida

Let G be a graph on n vertices with independence number α. Hadwiger’s Conjecture implies that G contains a clique minor of size at least n/α. In this talk, I will survey recent progress on this problem and present some new results.

On Some Chi-Bounded Classes of Graphs

Stéphan Thomassé, Ecole Normale Supérieure de Lyon, France

Planarity and Dimension for Graphs and Posets

William T. Trotter, Georgia Institute of Technology

There is a rich history of research relating planarity for graphs and diagrams with the dimension of posets, starting with the elegant characterization of planarity for posets with a zero and a one: they are planar if and only if they have dimension at most 2. Planar posets with a zero (or a one) have dimension at most 3, but Kelly showed that there are planar posets of arbitrarily large dimension. Subsequently, Schnyder proved that a graph is planar if and only if the dimension of its incidence poset is at most 3. Quite recently, Felsner, Wiechert and Trotter have shown that the dimension of a poset with a planar comparability graph is at most 4, while Streib and Trotter have shown that the dimension of a poset with a planar cover graph is bounded as a function of its height.

A Colin de Verdiere-Type Invariant and Odd-K_4- and Odd-K_3^2-Free Signed Graphs

Hein Van der Holst, Georgia State University

A signed graph is a pair (G, Σ) where G is an undirected graph and $\Sigma \subseteq E(G)$. The edges in Σ are called odd and the other edges are called even. If $U \subseteq V(G)$, then re-signing (G, Σ) on U gives the signed graph $(G, \Sigma \Delta \delta(U))$. A signed graph is a minor of (G, Σ) if it comes from (G, Σ) by a series of re-signing, deletion of an edge, and contraction of an even edge.

We introduced a new Colin de Verdiere-type invariant $\nu(G, \Sigma)$ for signed graphs. This invariant is closed under taking minors, and characterizes bipartite signed graphs as those signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 1$, and signed graphs with no odd-K_4- and no odd-K_3^2-minor as those signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 2$. In this talk we will discuss this invariant and these results.

Joint work with Marina Arav, Frank Hall, and Zhongshan Li.

Braces and Pfaffian Orientations

Peter Whalen, Georgia Institute of Technology

Robertson, Seymour, Thomas and simultaneously McCuaig answered several equivalent questions. Specifically, when can some of the 1's of a 0-1 square matrix, A, be changed to -1's so that the permanent of A equals the determinant of the modified matrix? When is a hypergraph with n vertices and n hyperedges minimally nonbipartite? When does a bipartite graph have a Pfaffian orientation? Given a digraph, does it have an even directed circuit? When is a square matrix sign non-singular? We provide a much shorter proof using elementary methods for their theorem. This is joint work with Robin Thomas.

Tangles, Trees and Flowers

Geoff Whittle, Victoria University of Wellington, New Zealand

Tangles provide a means of identifying highly connected components of a graph, matroid or, more generally, a connectivity function. For a tangle of order k in such a structure, the smallest order of a separation that can cross the structure in a non-trivial way relative to the tangle has order k. It turns out that, modulo a natural notion of equivalence, such k-separations do not behave arbitrarily. Indeed there is a tree that describes, up to equivalence, all such k-separations. This tree generalises known tree decompositions for 2- and 3-separations in connected and 3-connected matroids and graphs. This is joint work with Ben Clark.

Excluding a Clique Immersion

Paul Wollan, University of Rome "La Sapienza", Italy

We present an easy structure theorem for graphs which do not admit an immersion of the complete graph K_t. The theorem motivates the definition of a variation of tree decompositions based on edge cuts instead of vertex cuts which we call tree-cut decompositions. We give a definition for the width of tree-cut decompositions, and using this definition along with the structure theorem for excluded clique immersions, we prove that every graph either has bounded tree-cut width or admits an immersion of a large wall.

This is joint work with Paul Seymour.

Progress on Steinberg's Conjecture: History and Recent Results

Carl Yerger, Davidson College

A famous conjecture of Steinberg states that if a planar graph excludes 4-cycles and 5-cycles, it is 3-colorable. In this talk, we will give a description of important papers related to this conjecture and give a sense of the proof techniques involved. In addition we will describe recent work of Thomas and Yerger that give new related results for graphs on higher surfaces. In particular, if graph G is drawn in surface S, is 4-critical and has no cycles of length four through ten, then the number of vertices in G is at most a constant, c, times the Euler genus of S, where c is an explicit constant that comes out of the proof.

Posters

Matching and Small Paths in Fullerene Graphs

Afshin Behmaram, University of Illinois, Chicago

Fullerene graph is 3-regular planar graph which faces is pentagon or Hexagon. The number of matching in fullerene graph has some apllication in Chemistry and nanotube. We found a formula for the number of small Paths in Fullerene Graphs.by using this formula, we count the number of small matching and independent set in fullerene graph. Also, we find upper bound for the number of perfect matching in (3,6) fullerene.

The Cutting Plane Method is Polynomial for Perfect Matchings

Karthekeyan Chandrasekaran, Georgia Institute of Technology

We prove that the cutting plane approach using Edmonds' blossom inequalities converges in polynomial time for the minimum-cost erfect matching problem. The cutting plane approach to optimal matchings has been discussed by several authors over the past decades [Padberg-Rao, Lovasz-Plummer, Grotschel-Holland, Trick, Fischetti-Lodi], and its convergence has been an open question. We also show that the intermediate linear programs have half-integral optima for our choice of cutting planes.

Recent Advances in Finite Planar Emulators

Martin Derka, Masaryk University, Czech Republic

The poster presents the recent development in the field of planar emulators of graphs. For a long time, it was believed that the class of graphs with finite planar emulators is the class of graphs with projective embedding. This conjecture was surprisingly disproved in 2008 when emulators for two minor-minimal obstructions for the projective plane were published. We present emulators for other non-projective graphs as well as metodology for our exhaustive computations attempting to provide new characterization of the class of planar-emulable graphs.

A Ramsey Theorem for Indecomposable Matchings

James Fairbanks, Georgia Institute of Technology

A matching is indecomposable if it does not contain a nontrivial contiguous segment of vertices whose neighbors are entirely contained in the segment. We prove a Ramsey-like result for indecomposable matchings, showing that every sufficiently long indecomposable matching contains a long indecomposable matching of one of three types: interleavings, broken nestings, and proper pin sequences.

Unavoidable Minors for Connected 2-Polymatroids

Dennis Hall, Louisiana State University

It is well known that, for any integer n greater than one, there is a number r such that every 2-connected simple graph with at least r edges has a minor isomorphic to an n-edge cycle or $K_{2,n}$. This result was extended to matroids by Lovasz, Schrijver, and Seymour who proved that every sufficiently large connected matroid has an n-element circuit or an n-element cocircuit as a minor. In this talk, we generalize these theorems by providing an analogous result for connected 2-polymatroids.

Nested Cycles in Large Triangulations and Crossing-Critical Graphs

Cesar Hernandez Velez, Universidad Autonoma de San Luis Potosi, Mexico

We show that every sufficiently large plane triangulation has a large collection of nested cycles that either are pairwise disjoint, or pairwise intersect in exactly one vertex, or pairwise intersect in exactly two vertices. We apply this result to show that for each fixed positive integer k, there are only finitely many k-crossing-critical simple graphs of average degree at least six. Combined with the recent constructions of crossing-critical graphs given by Bokal, this settles the question of for which numbers $q > 0$ there is an infinite family of k-crossing-critical simple graphs of average degree q.

Finding Light Spanners in Bounded Pathwidth Graphs

Hao-Hsiang Hung, Georgia Institute of Technology

Given an edge-weighted graph G and $\epsilon > 0$, a $(1+\epsilon)$-spanner is a spanning subgraph G' whose shortest path distances approximate those of G within a $(1+\epsilon)$ factor. If G is from certain minor-closed graph families (at least bounded genus graphs and apex graphs), then we know that light spanners exist. That is, we can compute a $(1+\epsilon)$-spanner G' with total edge weight at most a constant times the weight of a minimum spanning tree. This constant may depend on ϵ and the graph family, but not on the particular graph G nor on its edge weighting. For weighted graphs from several minor-closed graph families, the existence of light spanners has been essential in the design of approximation schemes for the metric TSP (the traveling salesman problem) and some similar problems. In this paper we make some progress towards the conjecture that light spanners exist for every minor-closed graph family. In particular, we show that they exist for graphs with bounded pathwidth. We do this via the construction of light enough monotone spanning trees in such graphs.

Reed-Frost Transition on Dense Graphs

Nana Li, Georgia State University

In this paper, we consider a generalization of the classic Reed-Frost stochastic epidemic process by replacing the underlying complete graphs to dense graphs. Let $p \in (0,1)$ be a real number, G be a graph and $I_0 \subset V(G)$ be a vertex set of G. Let $(G, p, |I_0|)$ denote the *dynamic discrete percolation process* starting with the initial active vertex (infected individual) set I_0 such that all infected individuals *infect* their susceptible neighbors independently with probability p at each time step during the former's infectious time period; equivalently, a susceptible individual gets infected with probability $1-(1-p)^{d_{I_t}(v)}$ at time step $t+1$, where I_t is the set of newly infected individuals during time step t and $d_{I_t}(v)$ is the number of v's active neighbors in I_t. Additionally, all vertices in I_t are assumed to be removed by time step $t+1$. The process stops the first time $I_t = \emptyset$. Let $A_t = \cup_{i=0}^{t} I_i$. Then, as time t progresses, we obtain an increasing sequence of removed vertex sets $A_0 \subseteq A_1 \subseteq A_2 \subseteq \cdots \subseteq A_t \subseteq \cdots$. The *percolation time* $t(G, p, I_0)$ is the least T with $A_T = V(G)$ if such a T exists or ∞ otherwise. Given three real numbers $a, b, b' \in (0, 1)$ with $b' > b$, we show that there exists a T, depending only on a, b, b', such that with high probability, for any graph G of order n with minimum degree at least an and edge-connectivity at least $n^{b'}$, the percolation time $t(G, 1/n^b, I_0) \leq T$ even for $|I_0| = 1$. We also show that setting the probability to be $1/n^b$ and minimum degree to be at least αn is the best possible. By picking each vertex in I_0 uniformly and randomly with probability fq_n satisfying $\lim_{n\to\infty} nfq_n = \infty$, we show a similar result for graphs of order n which only requires minimum degree at least an.

Sampling 3-Orientations of Triangulations

Sarah Miracle, Georgia Institute of Technology

Given a planar triangulation, a 3-orientation is an orientation of the internal edges so all internal vertices have out-degree three. Each 3-orientation gives rise to a unique edge coloring known as a Schnyder wood that has proven useful for various computing and combinatorics applications. In joint work with Dana Randall, Amanda Streib and Prasad Tetali, we consider natural Markov chains for sampling uniformly from the set of 3-orientations. First, we study a "triangle-reversing" chain on the space of 3-orientations of a fixed triangulation that reverses the orientation of the edges around a triangle in each move. We show that (i) when restricted to planar triangulations of maximum degree six, the Markov chain is rapidly mixing, and (ii) there exists a triangulation with high degree on which this Markov chain mixes slowly. Next, we consider an "edge-flipping" chain on the larger state space consisting of 3-orientations of all planar triangulations on a fixed number of vertices. It was also shown previously that this chain connects the state space and we prove that the chain is always rapidly mixing.

Homeomorphically Irreducible Spanning Trees

Songling Shan, Georgia State University

A spanning tree T of a graph G is called a *homeomorphically irreducible spanning tree* (HIST) if T does not contain vertices of degree 2. In 1974, Hill conjectured that every triangulation of the plane contains a HIST. In 1990, Abertson, Berman, Hutchinson, and Thomassen confirmed the conjecture, and asked if every triangulation of a surface contains a HIST and more generally if a graph with every edge in at least two triangles contains a HIST. In 2009, those questions were restated by Archdeacon as conjectures.

We confirm the first conjecture by showing that every connected and locally connected graph with more than 3 vertices contains a HIST, and confirm the second one by constructing spanning subgraphs of graphs with every edge contained in two triangles and showing the spanning subgraph contains a HIST.

On a Class of Nearly Binary Matroids

Jesse Taylor, Louisiana State University

We give an excluded-minor characterization for the class of matroids M in which $M\backslash e$ or M/e is binary for all e in $E(M)$.